

# **Dynamical Systems & Nonlinear Partial Differential Equations**

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## **1. Technical Objectives and Motivation**

The research conducted during the past three years is part of a long term project whose objective is the study of certain nonlinear differential equations and dynamical systems that model significant physical phenomena. Five principal investigators are involved, together with their visitors and students. Consequently one may distinguish several directions in the research: problems in the area of hyperbolic systems of conservation laws pertaining to stability issues in Continuum Physics; questions arising in the study of dynamical systems generated by functional differential equations; stability and instability of solutions of evolution equations of mathematical physics.

## **2. Approach**

Despite the diversity of the projects, there is a certain degree of unity induced by the interactions between the PI's as well as by their shared "philosophy" of employing ideas from the theory of dynamical systems in the study of partial differential equations and functional differential equations.

## **3. Significant Accomplishments**

In the area of hyperbolic systems of conservation laws (Dafermos):

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- Stability of solutions of systems of conservation laws in several space dimensions.

In the area of dynamical systems (Jones, Mallet-Paret):

- Stability of travelling waves with applications in nonlinear optics, pattern formation and nerve impulse propagation.
- Study of lattice differential equations and related areas.

In the area of equations of mathematical physics (Craig, Strauss):

- Instability of stationary equilibria for Vlasov-Poisson systems.
- Dispersive smoothing for the nonlinear Schrödinger equation.
- Normal form for the water waves problem.

#### 4. Selected Refereed Publications

C. Dafermos, "Stability for systems of conservation laws in several space dimensions", *SIAM J. Math. Analysis* **26** (1995), 1403-1414.

C. Dafermos, "A system of conservation laws with frictional damping", *Zeit. Ang. Math. Phys.* **46** (1995), S294-S307.

C. Dafermos, "The vanishing viscosity method in one-dimensional thermoelasticity", (with G.-Q. Chen), *Trans. AMS* **347** (1995), 531-541.

C. Dafermos, "Hyperbolic systems of conservation laws", *Proceedings Int. Congress Math. 1994*, Birkhäuser Verlag, Basel (1995), 1096-1107.

W. Craig & P. Worfolk, "An integrable normal form for water-waves in infinite depth" (1995) *Physica D* **84**, p513.

W. Craig, T. Kappeler & W. Strauss, "Microlocal dispersive smoothing for Schrödinger's equation", *Commun. Pure Applied Math.* **48**, (1995), in press.

C.K.R.T. Jones, "Stability of the in-phase travelling wave solution in a pair of coupled nerve fibers", *Indiana University Math. J.* **44** (1995) 189-220 (with A. Bosc).

C.K.R.T. Jones, "Tracking invariant manifolds up to exponentially small errors", *SIAM J. Math. Anal.* (in press) (with T. Kaper and N. Kopell).

C.K.R.T. Jones, "Settling and asymptotic motion of aerosol particles in a cellular flow field" *J. Nonlinear Science* 4 (1995) 337-358 (with M. Maxey and J. Rubin).

C.K.R.T. Jones, "Chaotic mixing across oceanic jets" *Proc. ONR/NUWC Conference on Nonlinear Signal Processing*, Mystic, CT, July 1995 (with P. Miller, G. Haller and L. Pratt).

J. Mallet-Paret & S.N. Chow, "Pattern formation and spatial chaos in lattice dynamical systems", *IEEE Trans. Circuits Syst.* 42 (1995), 746-751, 752-756.

J. Mallet-Paret, "Spatial patterns, spatial chaos, and traveling waves in lattice differential equations, in: *Stochastic and Spatial Structures of Dynamical Systems* (eds. S.J. van Strien and S.M. Verduyn Lunel), North-Holland, Amsterdam, 1996, pp. 105-129.

J. Mallet-Paret, "Pattern formation and spatial chaos discrete evolution equations", (with S.N. Chow and E. Van Vleck), *Random & Computational Dynamics* 4 (1996), 109-178.

J. Mallet-Paret, "Dynamics of lattice differential equations", (with S.N. Chow and E. Van Vleck), *Int. J. Bifurcation and Chaos* (1996).

W. Strauss & Y. Guo, "Nonlinear instability of double-humped equilibria", *Annales IHP (Anal. Non Lin.)* 12 (1995), 339-352.

W. Strauss, R. Glassey, "Asymptotic stability of the relativistic maxwellian via 14 moments", *Transp. Th. Stat. Physics* 24 (1995), 657-678.

## 5. Papers or reports in non-refereed publications

C. Dafermos, "A system of conservation laws with frictional damping", *Zeit. Ang. Math. Phys.* 46 (1995), S294-S307.

C. Dafermos, "The vanishing viscosity method in one-dimensional thermoelasticity", (with G.-Q. Chen), *Trans. AMS* 347 (1995), 531-541.

**C. Dafermos**, "Entropy for hyperbolic systems of conservation laws in several space dimensions" (to appear).

**C. Dafermos**, "Global solutions for a system of conservation laws in viscoelasticity", (with G.-Q. Chen) (to appear).

**C. Dafermos**, "Entropy and the stability of classical solutions of hyperbolic systems of conservation laws" (to appear).

**C.K.R.T. Jones**, "Stability of pulses on optical fibers with phase-sensitive amplifiers", submitted (with J. Alexander, M. Grillakis and B. Sandstede).

**C.K.R.T. Jones**, "Phase field models for hypercooled solidification", submitted (with P. Bate, P. Fife and R. Gardner).

**C.K.R.T. Jones**, "The existence of travelling wave solutions of a generalized phase-field model" to appear in SIAM J. Math. Anal. (with P. Bates, P. Fife and R. Gardner).

**C.K.R.T. Jones**, "Existence and stability of  $N$ -pulses on optical fibers with phase-sensitive amplifiers" to appear in Physica D (with B. Sandstede and J. Alexander).

**J. Mallet-Paret**, "Traveling wave solutions for systems of ODE's on a two-dimensional spatial lattice (with J.W. Cahn and E. Van Vleck), submitted for publication.

**J. Mallet-Paret**, "Boundary Layer Phenomena for Differential-Delay Equations with State Dependent Time Lags: III (with R. Nussbaum), in preparation.

**J. Mallet-Paret**, "Traveling waves in lattice dynamical systems, (with S.N. Chow and W. Shen), in preparation.

**J. Mallet-Paret**, "The Fredholm alternative for functional differential equations of mixed type, in preparation.

**J. Mallet-Paret**, "Normal Forms for Singularly Perturbed Vector Fields (with S.N. Chow and P. Brunovsky), in preparation.

**J. Mallet-Paret**, "Spatial Entropy of Stable Mosaic Solutions for a Class of Spatially Discrete Evolution Equations (with S.N. Chow and E. Van Vleck), in preparation.

**W. Strauss, S. Friedlander and M. Vishik**, "Nonlinear instability in an ideal fluid", Annales IHP (*Anal. Non Lin.*), to appear.

**W. Strauss**, "Stability, instability and regularity of nonlinear waves, in: Nonlinear Waves, T. Ozawa, ed., Gakuto Int'l Series, Gakkotosho, Tokyo, *in press*.

**W. Strauss**, "Breathers as homoclinic geometric wave maps, *Physica D* (with J. Shatah), *in press*.

**W. Strauss**, "Existence and blow up of small-amplitude nonlinear waves with a negative potential, Disc. Cont. Dyn. Sys. (with K. Tsutaya), *in press*.

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB NO. 0704-0188

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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE	3. REPORT TYPE AND DATES COVERED Final Technical Report 5/1/93-10/31/96	
4. TITLE AND SUBTITLE Dynamical Systems & Nonlinear Partial Differential Equations			5. FUNDING NUMBERS DAAH04-93-G-0198	
6. AUTHOR(S) Constantine Dafermos John Mallet-Paret Walter Craig Walter Strauss Christopher Jones				
7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES) Division of Applied Mathematics Lefschetz Center for Dynamical Systems Brown University Providence, RI 02912			8. PERFORMING ORGANIZATION REPORT NUMBER 0-21232-1350	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSORING / MONITORING AGENCY REPORT NUMBER ARO 30775.18-MA	
11. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12 b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  The research conducted during the past three years is part of a long term project whose objective is the study of certain nonlinear differential equations and dynamical systems that model significant physical phenomena. Five principal investigators are involved, together with their visitors and students. Consequently one may distinguish several directions in the research: problems in the area of hyperbolic systems of conservation laws pertaining to stability issues in Continuum Physics; questions arising in the study of dynamical systems generated by functional differential equations; stability and instability of solutions of evolution equations of mathematical physics.				
14. SUBJECT TERMS			15. NUMBER OF PAGES 5	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OR REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

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